MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

DIM5068 – MATHEMATICAL TECHNIQUES 2

(for DIT students only.)

19 OCTOBER 2016
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 5 pages with 5 questions only.
2. Attempt **ALL FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Write your answers in the answer booklet provided.
4. Formulae are provided in the appendix section.

QUESTION 1

a. Find the values of x and y in the equation $x(1+i)^2 + y(2-i)^2 = 3+10i$. (6 marks)

b. Given $z = -3+4i$ and $zw = -14+2i$,

i. Find w in the form of $a+bi$. (5 marks)

ii. Find the modulus and the argument of w . (4 marks)

c. Solve the equation $2x^2 - 2ix - 5 = 0$ and give the answer in the form of $a+bi$. (5 marks)

[TOTAL 20 MARKS]

QUESTION 2

a. Given the function $y = x^3 \cos x$, find $\frac{dy}{dx}$. (3 marks)

b. Differentiate $3y^2 - 2x^2 = 2xy$ by using implicit differentiation. (6 marks)

c. The volume, $V\ m^3$, of water in a tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2 \quad , \quad for\ t \geq 0.$$

i. Find $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$. (4 marks)

ii. Find the rate of change of the volume of water in the tank, in $m^3 s^{-1}$, when $t = 2$. (2 marks)

iii. Verify that V has a stationary value when $t = 1$. (2 marks)

iv. Determine whether value V is a maximum or minimum value. (3 marks)

[TOTAL 20 MARKS]

Continued...

QUESTION 3

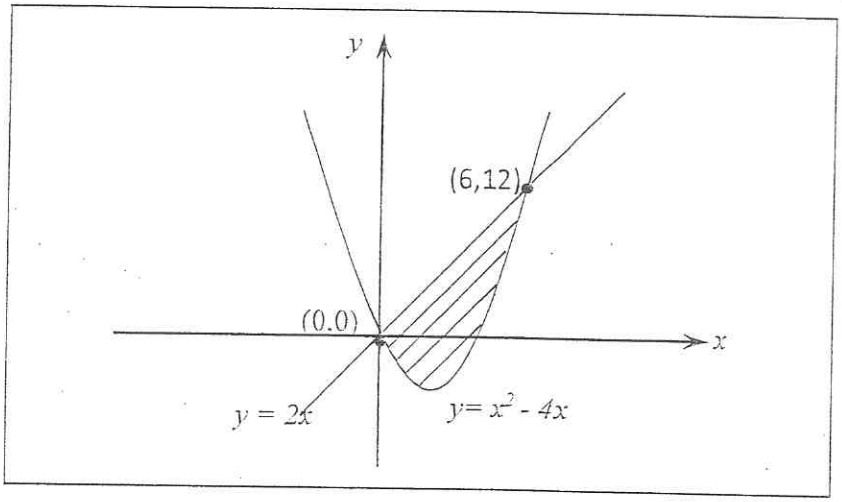
a. Find the following integrals.

i. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(8y^2 + \sin y\right) dy$ (5 marks)

ii. $\int (x^3 - 2)^6 (3x^2)\, dx$ [Hint: use **Substitution Rule**] (4 marks)

iii. $\int_0^2 (x-2)e^x dx$ [Hint: use **Integration By Part**] (6 marks)

b. Find the area of region bounded by the graphs of $y = x^2 - 4x$, $y = 2x$, $x = 0$ and $x = 1$. (5 marks)



[TOTAL 20 MARKS]

QUESTION 4

a. Solve the differential equation $e^x \frac{dy}{dx} = 2x$ by using separation of variables. (6 marks)

b. Use the integrating factor to find the general solution of (7 marks)

$$x\frac{dy}{dx} - 2y = 10x^3 .$$

c. Find the solution of the initial value problem $y'' - 4y' + 4y = 0$ if $y(0) = 3$ and $y'(0) = 1$. (7 marks)

[TOTAL 20 MARKS]

Continued...

QUESTION 5

a. Given $\vec{a} = \langle -3,2,1 \rangle$ and $\vec{b} = \langle 2,0,-2 \rangle$, find

i. $2\vec{a} + 4\vec{b}$ (2 marks)

ii. $|2\vec{a} + 4\vec{b}|$ (2 marks)

b. Given $\vec{u} = -3\vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{v} = 6\vec{i} + 3\vec{j} + \vec{k}$. Find the cross product for this two vectors. (4 marks)

c. Find the angle between the vectors $\vec{a} = \langle 2,2,-1 \rangle$ and $\vec{b} = \langle 5,-3,2 \rangle$. (8 marks)

d. Find the parametric equation of the line passing through the point $P(1,3,-2)$ and parallel to vector $\vec{v} = \overrightarrow{5i} - \overrightarrow{7j} + \overrightarrow{3k}$. (4 marks)

[TOTAL 20 MARKS]

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APPENDIX

Derivatives: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(a^x) = a^x \ln a$
3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
4. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Table of Integrals

1. $\int u\, dv = uv - \int v\, du$
2. $\int u^n\, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u du = e^u + C$
5. $\int \sin u\, du = -\cos u + C$
6. $\int \cos u\, du = \sin u + C$
7. $\int \sec^2 u\, du = \tan u + C$
8. $\int \csc^2 u\, du = -\cot u + C$
9. $\int \sec u \tan u\, du = \sec u + C$
10. $\int \csc u \cot u\, du = -\csc u + C$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)]\, dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left|\vec{PQ} \times \vec{PR}\right|$

Area for triangle PQR

$= \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\cos\theta = \dfrac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$